Stratified Turbulent-Turbulent Gas-Liquid Flow

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This paper is an experimental and theoretical study of horizontal stratified gas-liquid two phase flow in a circular tube. Both phases are considered to be in turbulent flow, and the liquid phase flow field is modeled by applying eddy viscosity expressions developed for single phase flow. The pressure drop and in situ volume of liquid are predicted from the gas and liquid flow rates, physical properties, and pipe size by means of an iterative procedure which terminates when calculated gas and liquid pressure drops match. The iterative design procedure is compared with new data for air-water flow in a smooth tube of 63.5 mm ID and with data available in the literature. For conditions corresponding to small amplitude interfacial waves, the average deviation between predicted and experimental results is 24.3% for the pressure drop and 7.7% for the holdup. For roll wave conditions, the corresponding average deviations are 4.6 and 26.4% for pressure drop and holdup, respectively. These results are substantially better than the predictions obtained using the Lockhart-Martinelli correlations.

SCOPE

The prediction of the flow characteristics (primarily pressure drop and holdup) for two phase gas-liquid flows has been based largely on empirical correlations, the most widely used being that of Lockhart and Martinelli (1949). Several modifications of this correlation have been proposed (Bertuzzi et al., 1956; Baroczy, 1965; and Hoogendoorn, 1959), but errors of 50% and more have been reported in their application. The principal defect in such empirical correlations is that details of the flow structure other than the flow pattern are not incorporated, and the results are not readily adapted to predict heat and mass transfer characteristics with two phase flows.

More recently, mechanistic approaches have been used successfully to predict pressure drop, holdup, and other fluid mechanics parameters for horizontal stratified gas-liquid flows in pipelines and conduits. Yu and Sparrow (1967) treated laminar/laminar two phase flows analytically, and they obtained closed form solutions relating volumetric flow rates, pressure drop, and holdup for special limiting cases. Etchells (1970), Govier and Aziz (1972), and Agrawal et al. (1973) proposed calculation of the frictional pressure drop for laminar liquid/turbulent gas stratified flow from friction factors based on single phase flow and a geometrical model of the flow cross section. Russell et al. (1974) modified this approach by incorporating an approximate solution of the equation of motion for the liquid phase, and the resulting design procedure was found to agree well with pressure drop and holdup data for air-water and air-glycerine stratified flows in tubes of 25.4, 38.1, and 50.8 mm ID.

All of the mathematical models for laminar/laminar and laminar/turbulent gas-liquid flows require an iterative procedure to predict pressure drop and holdup from knowledge of the flow rates, physical properties, and pipe diameter, but the results are superior to the Lockhart-Martinelli method.

It is the object of this study to develop an analysis and design procedure for turbulent/turbulent gas-liquid stratified flow and to compare predicted pressure drop and holdup results with new experimental data and data available in the literature.

The analysis represents an extension of the model of Russell et al. (1974) to a turbulent liquid phase using single phase flow turbulence concepts that have been applied successfully to vertical, annular, two phase flow. Dukler (1960) used such an approach for concurrent downflow in vertical tubes, and Hewitt (1961) applied Dukler's method to concurrent gas-liquid upflow. The principal advantage of this type of modeling is that not only the fluid flow characteristics can be predicted but also the heat transfer characteristics by applying analogies between momentum transfer and heat transfer.

CONCLUSIONS AND SIGNIFICANCE

An analysis and design procedure have been developed and tested to predict pressure drop and holdup for horizontal stratified gas-liquid flow in pipelines for turbulent liquid/turbulent gas conditions. The iterative procedure developed is an extension of previously proposed methods using an improved model of the liquid phase flow field which incorporates eddy viscosity expressions based on single phase flow in pipes.

Experiments were performed using air-water flow in a tube of 63.5 mm ID. The in situ volume of liquid and interfacial wave structure were measured by electrical conductance probes, and other flow parameters were measured by conventional means. For 62 data points involving small amplitude interfacial waves, the predicted pressure drop has an average absolute deviation from experiment of 24.3%, and the holdup predicted has an average deviation of 7.7%. For roll waves, 111 data points show an average deviation of only 4.6% between predicted and experimental pressure drops, and 73 data points for the holdup show an average deviation of 26.4%. For these conditions, the Lockhart-Martinelli correlation is in error by as much as 100% at higher liquid volume fractions. The results of Jensen (1972) and Arruda (1970) that are near the turbulent/turbulent regime are consistent with our data, and the predicted pressure drops and holdup are in agreement with their measurements.

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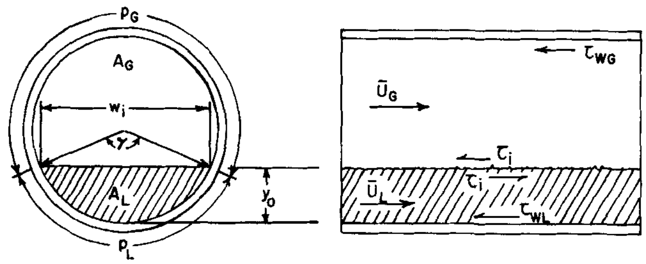

Fig. 1. The stratified flow system under consideration.

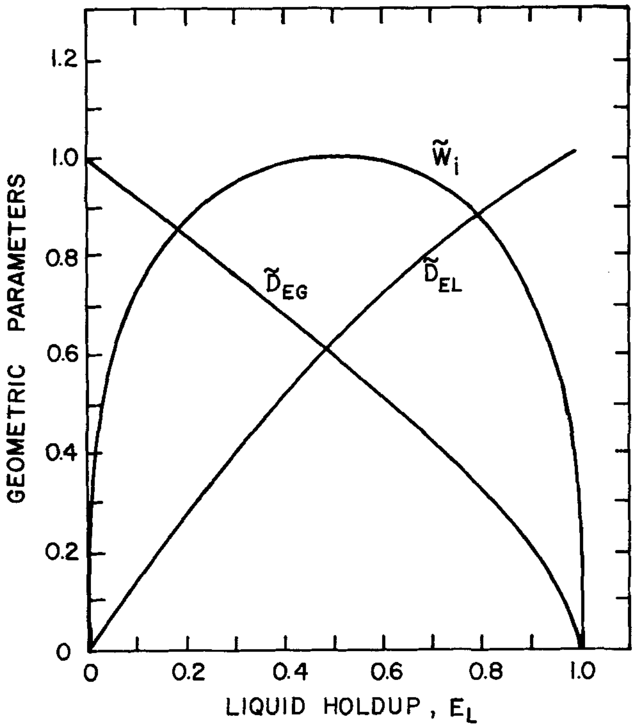

Fig. 2. The geometric parameters of Fig. 1 as a function of the in situ liquid volume fraction.

THE MATHEMATICAL MODEL

The in situ volume or holdup of the liquid is related by geometry to the distance y_o between the gas-liquid interface and the lowest point of the pipe and to other parameters of the system. Considering the geometry shown in Figure 1 and following Govier and Aziz (1972), we may write the geometrical relations

$$y_o/R = 1 - \cos\gamma/2 \tag{1}$$

$$\tilde{w}_i = w_i/D = 2[y_o/D) - (y_o/D)^2]^{1/2} \tag{2}$$

$$E_L = A_L/A = (1/2\pi)(\gamma - \sin\gamma) \tag{3}$$

$$E_G = A_G/A = 1 - E_L \tag{4}$$

$$\tilde{p}_L = p_L/p = \gamma/2\pi = 1 - p_G/p = 1 - \tilde{p}_G \tag{5}$$

$$\tilde{D}_{EL} = D_{EL}/D = 4A_L/P_LD = E_L/(p_L/p) \tag{6}$$

$$\tilde{D}_{EG} = D_{EG}/D = 4A_G/(p_G + w_i)D$$
$$= E_G/[(p_G/p) + (w_i/\pi D)] \tag{7}$$

The parameters w_i/D, D_{EL}/D, D_{EG}/D, p_G/p, and p_L/p are plotted as functions of E_L in Figures 2 and 3.

We should point out that the gas-liquid interface is not usually smooth when the two phases are in turbulent flow, so we shall consider y_o to be the average distance from the interface to the bottom of the pipe; that is, we shall take $y_o = \bar{\delta}$, where $\bar{\delta}$ is the average liquid depth on a vertical plane through the tube center line. For small amplitude waves such as shown in Figure 4, the average depth can be estimated directly from the oscillograph tracings, but for the large amplitude waves (roll waves) shown in the figure, we shall apply the simple averaging technique of Permyakov and Podsushnyy (1971), who employed the expression

$$\bar{\delta} = \frac{\delta_{max} + \delta_c + 2\delta_{min}}{4} \tag{8}$$

Let y be the distance from the tube wall in the direction of the inward normal to the wall. Then the position of the interface is given by

$$y_i/R = 1 - (\cos\gamma/2)(1 + \tan\theta)^{1/2} \tag{9}$$

where $\theta = \cos^{-1}(R - y_o/R - y_i)$.

For a turbulent liquid, the momentum flux or shear stress can be expressed by

$$\tau = \rho_L(\nu_L + \epsilon_M)\frac{du}{dy} \tag{10}$$

The shear stress distribution τ is obtained by means of a force balance on a liquid layer extending from $y = 0$ to arbitrary position $y = y$. The result is

$$\tau = \frac{R}{2(R-y)\theta}\tau_{WL}$$
$$- \frac{(2Ry_o - y_o^2)^{1/2} - [(2Ry_o - y_o^2) - (2Ry_i - y_i^2)]^{1/2}\tau_i}{(R-y)\theta}$$
$$- \frac{(\Delta P/L)}{4}\left[\frac{R^2(\gamma - \sin\gamma)}{(R-y)\theta} - (R-y)\left(2 - \frac{\sin 2\theta}{\theta}\right)\right] \tag{11}$$

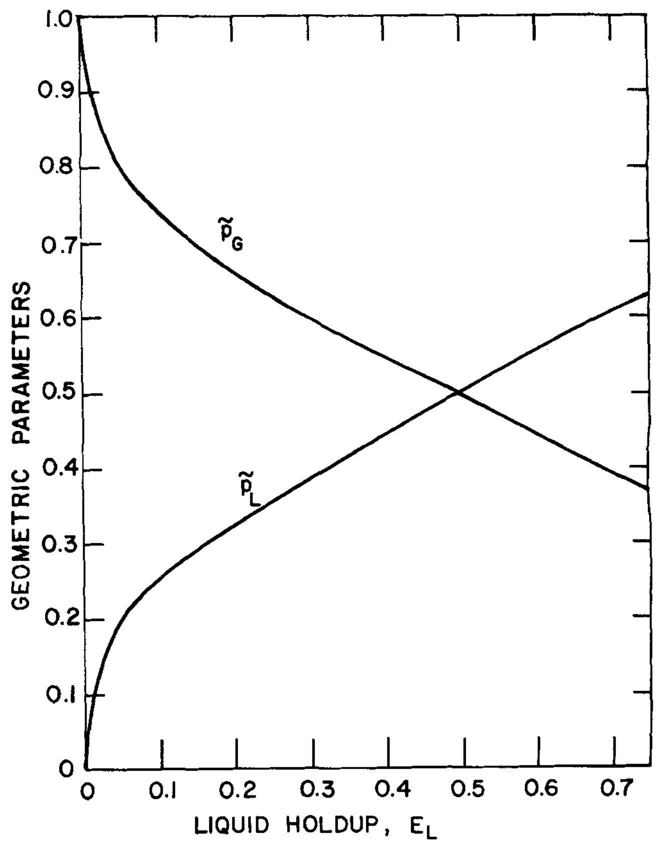

Fig. 3. The gas and liquid wetted perimeters as a function of holdup.

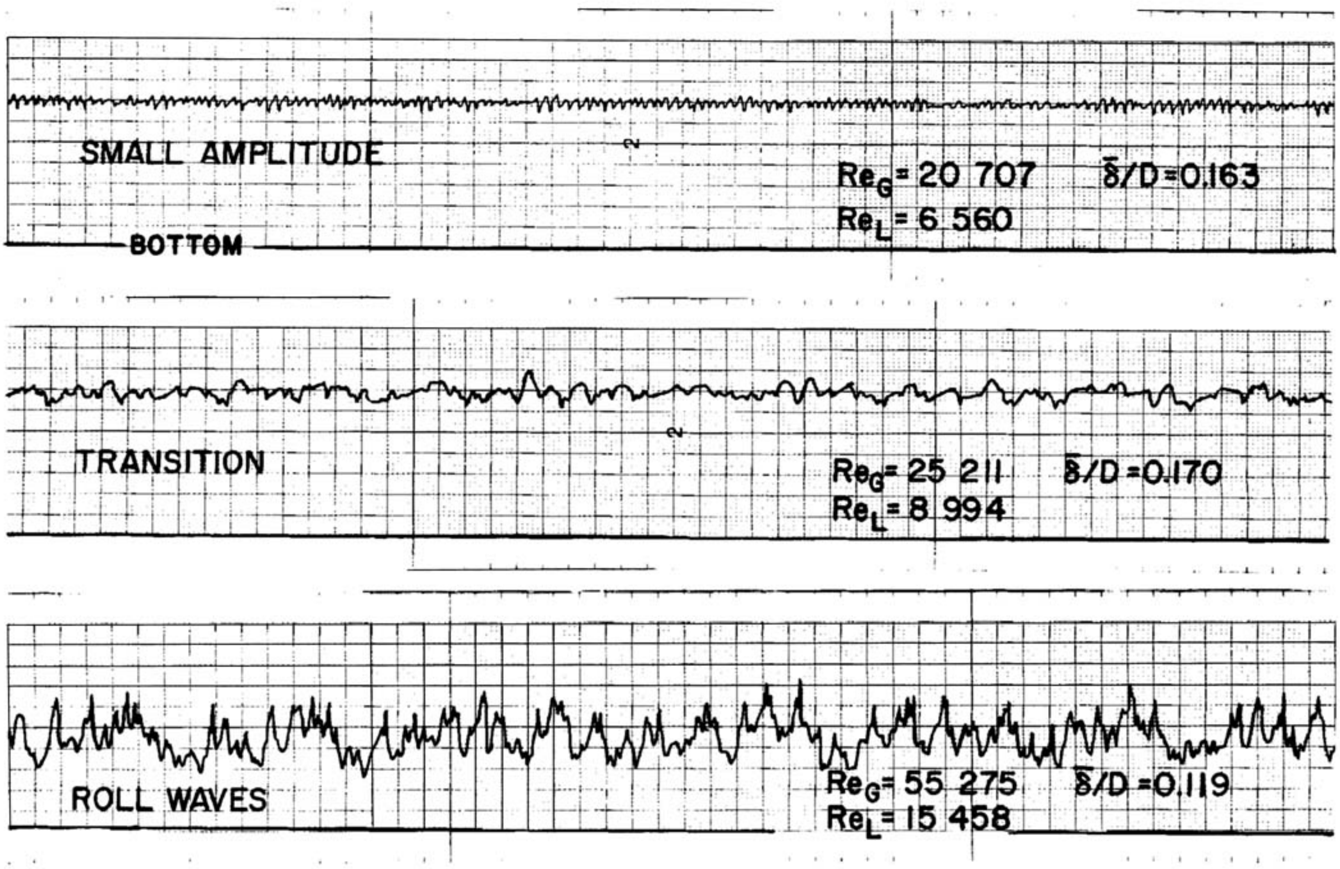

Fig. 4. Typical oscillograph tracings of the interfacial wave structure.

The interfacial shear τ_i and the wall shear τ_{WL} are related by

$$\tau_i = \tau_{WL}\frac{R\gamma}{w_i} - \left(\frac{\Delta P}{L}\right)\frac{R^2}{2w_i}(\gamma - \sin\gamma) \quad (12)$$

For small values of γ (small holdup), we may write the approximation

$$\tau \simeq \tau_{WL} \quad (13)$$

To greatly simplify the analysis, we shall assume this approximation to the valid over the whole range of holdup values encountered here. This can lead to some error at lower gas flow rates and high liquid flow rates for which the liquid occupies an appreciable fraction of the tube, but we shall justify this assumption a posteriori.

Let us introduce the usual dimensionless distance and velocity variables based on the friction velocity u^* as follows:

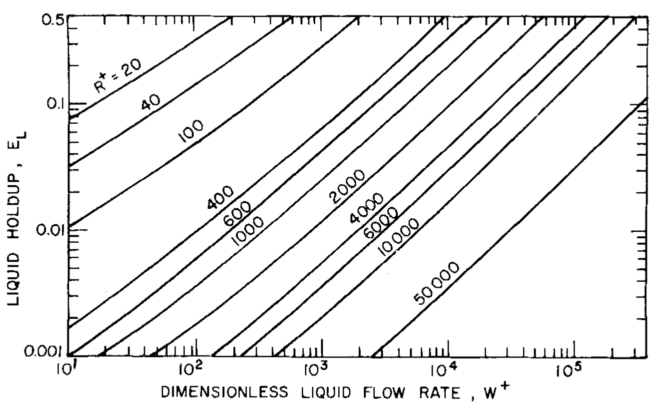

Fig. 5. The in situ liquid volume fraction as a function of dimensionless liquid flow rate and the dimensionless tube size parameter R^+.

$$u^* = (\tau_{WL}/\rho_L)^{1/2} \quad (14)$$

$$y^+ = u^* y/\nu_L \quad (15)$$

$$u^+ = u/u^* \quad (16)$$

We shall further assume that the turbulence characteristics of the liquid phase are similar to those for single phase flow for identical friction velocities; that is, we shall assume that single phase flow eddy viscosity expressions apply to the liquid phase. For the region near the wall ($0 \leq y^+ \leq 20$), we apply Deissler's (1959) equation

$$\epsilon_M = n^2 uy[1 - \exp(-n^2 uy/\nu_L)] \quad (17)$$

where $n = 0.1$, and for the turbulent core ($y^+ \geqq 20$) we shall use von Karman's (1939) expression

$$\epsilon_M = \chi^2 \frac{(du/dy)^3}{(d^2u/dy^2)^2} \quad (18)$$

where $\chi = 0.36$.

Combining Equations (10), (13), (15), (16), and (17), we get

$$\frac{\tau}{\tau_{WL}} \simeq 1 = 1 + n^2u^+y^+[1 - \exp(-n^2u^+y^+)]\frac{du^+}{dy^+} \quad \text{for} \quad 0 \leq y^+ \leq 20 \quad (19)$$

and for the turbulent core we obtain

$$\frac{\tau}{\tau_{WL}} \simeq 1 = \chi^2 \frac{(du^+/dy^+)^4}{(d^2u^+/dy^{+2})^2} \quad \text{for} \quad y^+ > 20 \quad (20)$$

Equation (19) has been solved numerically to obtain u^+ as a function of y^+, the well-known universal velocity profile. Equation (20) can be solved analytically using the conditions

$$\frac{du^+}{dy^+} \rightarrow \infty \quad \text{as} \quad y^+ \rightarrow 0$$

and $u^+ = u_{20}^+$ at $y^+ = y_{20}^+ = 20$, where u_{20}^+ is obtained from the solution of Equation (19). The result is

$$u^+ = u_{20}^+ + \frac{1}{2\chi} \ln \frac{y^+}{20} \tag{21}$$

We must now relate the velocity distribution to the total liquid flow rate by integrating over the area occupied by the liquid phase. In this way, we obtain the dimensionless flow rate W^+ as

$$W^+ \equiv \frac{W_T}{\mu_L R} = 2 \int_0^{\gamma/2} \int_0^{y_i^+} u^+(y^+)(1 - y^+/R^+) dy^+ d\theta \tag{22}$$

where

$$R^+ = u^* R/\nu_L$$

and y_i^+ is the position of the interface in nondimensional form.

Equation (22) has been integrated numerically, and the results are presented in Figure 5 as a plot of E_L vs. W^+ for various values of the parameter R^+. Provided that R^+ is known, the holdup can be determined from the total liquid flow rate using Figure 5. It remains to determine R^+, which requires the determination of τ_{WL}.

Measurements of the interfacial structure and the interfacial shear stress for air-water flow have been reported by Hanratty and Engen (1957), Lilleleht and Hanratty (1961), Ellis and Gay (1959), van Rossum (1959), Davis (1969), and Cohen and Hanratty (1968). The latter showed that for three-dimensional waves of relatively small amplitude, their interfacial shear stress measurements were correlated by defining a friction factor

$$\lambda_o = 8(u_o^*/\overline{u}_B)^2 \tag{23}$$

where $u_o^* = (\tau_i/\rho_G)^{1/2}$, and $\overline{u}_B$ is the average gas velocity taken over the space between the interface and the maximum in the velocity profile of the gas phase. In the three-dimensional wave regime λ_o was found to be constant; that is, the three-dimensional waves showed a behavior similar to fully rough solid surfaces with λ_o dependent primarily on the wave height. Typically, $\lambda_o = 0.057$ for their data. If we use this asymptotic limit for small amplitude waves and let $\overline{u}_B = \overline{U}_G = Q_G/A_G$, the interfacial shear becomes

$$\tau_i = 0.00355(\rho_G \overline{U}_G^2/2) \tag{24}$$

For roll waves, Miya, Woodmansee, and Hanratty (1971) represented the interfacial stress by the equation

$$\tau_i = f_i \rho_G (\overline{U}_G - c)^2/2 \tag{25}$$

They found f_i to be a linear function of the liquid Reynolds number $Re_L = \Gamma_L/\nu_L$. The data of Cohen and Hanratty (1968) are correlated by the equation

$$f_i = 0.0080 + 2.00 \times 10^{-5} Re_L \tag{26}$$

for $100 \leq Re_L \leq 1\,700$. Assuming $\overline{U}_G >> c$, we shall use Equation (25) with $c = 0$ for the roll wave regime.

It should be noted that Equation (26) is based on a limited range of Re_L, well below the conditions of this study (see Table 1); however, as will be shown, the correlation provides reasonable estimates of f_i for the air/water system.

By means of force balances on the gas and liquid phases, assuming no acceleration, we can write

$$A_G(\Delta P/L) = \tau_{WG} p_G + \tau_i w_i \tag{27}$$

and

$$A_L(\Delta P/L) = \tau_{WL} p_L - \tau_i w_i \tag{28}$$

The gas phase wall stress can be written in terms of the usual friction factor f_G to give

$$\tau_{WG} = f_G(\overline{U}_G^2 \rho_G/2) \tag{29}$$

where f_G is a function of the gas phase Reynolds number $Re_G = D_{EG}\overline{U}_G/\nu_G$, and $\overline{U}_G$ is based on the area occupied by the gas phase as indicated above.

In this study, a smooth tube was employed, so the conventional friction factor applicable is

$$f_G = 0.046\, Re_G^{-0.20} \tag{30}$$

Thus, τ_{WG} is obtained from Equations (29) and (30), and τ_{WG}, τ_{WL}, and $(\Delta P/L)$ must satisfy Equations (27) and (28) involving geometrical parameters A_G, A_L, p_G, p_L, and w_i. Equations (27) and (28) are the basis for a design procedure.

THE ITERATIVE PROCEDURE

An iterative procedure analogous to those proposed by Yu and Sparrow (1967) for laminar/laminar flow and Agrawal et al. (1973) and Russell et al. (1974) for laminar/turbulent flow can be used to determine the pressure drop and holdup from the gas and liquid flow rates, physical properties, and pipe size. The iterative solution can be incorporated into a design procedure by using flow regime maps such as the Baker-Scott (1963) flow map to establish if the flow is stratified. It must also be established that the flow is turbulent/turbulent by calculating the Reynolds numbers of the phases. The iterative procedure used to calculate the pressure drop and holdup for turbulent/turbulent stratified flow is:

1. Choose a trial value of the holdup E_L and calculate the geometric parameters D_{EG}, D_{EL}, p_G, p_L, and w_i for the tube diameter under consideration.
2. Calculate $W^+ = W_L/\mu_L R$ from the liquid mass flow rate W_L and determine R^+ by interpolation from Figure 5.
3. Calculate the friction velocity and then τ_{WL} from R^+ and compute τ_i and τ_{WG} from the appropriate equations given above.
4. Solve Equations (27) and (28) separately for $(\Delta P/L)$. If the two calculated values do not agree to within some specified accuracy, say $\pm 1\%$, a new value of E_L is assumed, and the procedure is repeated.

This procedure is greatly facilitated by starting the iterations with a reliable estimate of E_L. One approximation is to use the Lockhart-Martinelli correlation for holdup, which correlates E_L with the parameter X defined by

$$X^2 = (\Delta P/L)_L{}^S/(\Delta P/L)_G{}^S \tag{31}$$

Table 1. Range of Experimental Conditions

Re_L	Re_G	Q_L (m³/s) × 10^4	Q_G (m³/s) × 10^2	Liquid holdup R_L	$(\Delta P/L)_{tp}$ (N/m³)
5 000-18 000	11 000-50 000	0.5-2.9	0.8-8.0	0.05-0.27	5.0-270.0
$u_G{}^s$	$u_L{}^s$	W^+	$\overline{\delta}/D$	Pr_L	Pr_G
300-3 000	2.0-7.0	1 650-100 000	0.01-0.27	~7.0	~0.7

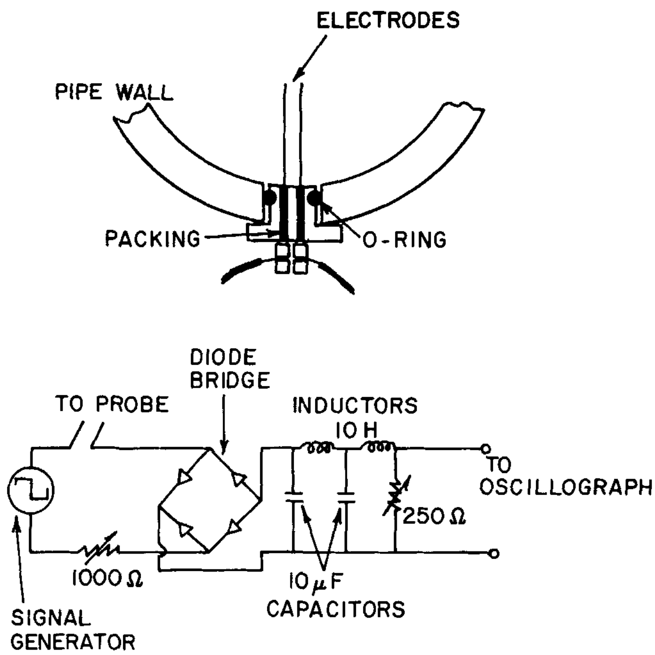

Fig. 6. Details of the parallel wire conductance probes and electrical circuit.

Because the Lockhart-Martinelli correlation for holdup is poor for stratified flow, an improved version of it is desirable, and it can be improved by incorporating the geometrical parameters discussed above as follows.

Eliminating the pressure drop from Equations (27) and (28), we get

$$\tau_{WG}\frac{\rho_G\overline{U}_G^2}{2}\frac{p_G}{A_G} - \tau_{WL}\frac{p_L}{A_L} + \tau_i w_i\left(\frac{1}{A_G}+\frac{1}{A_L}\right) = 0 \tag{32}$$

Approximating $\tau_i = \tau_{WG}$ and evaluating τ_{WL} and τ_{WG} in the conventional manner, that is, in terms of friction factors, we can write

$$\tau_{WL} = f_L\rho_L\overline{U}_L^2/2 \tag{33}$$

and

$$\tau_{WG} = f_G\rho_G\overline{U}_G^2/2 \tag{34}$$

with

$$f_L = C_L\left(\frac{D_{EL}\overline{U}_L}{\nu_L}\right)^{-m} \tag{35}$$

and

$$f_G = C_G\left(\frac{D_{EG}\overline{U}_G}{\nu_G}\right)^{-q} \tag{36}$$

Since the liquid and gas phases are both considered to be turbulent, we choose $C_L = C_G = 0.046$ and $m = q = 0.20$.

It is convenient to write Equation (32) in dimensionless form, denoting dimensionless quantities by a tilde (~). In this way, there results

$$X^2(\widetilde{D}_{EL}\widetilde{U}_L)^{-m}\widetilde{U}_L^2\frac{\widetilde{p}_L}{\pi E_L} - (\widetilde{D}_{EG}\widetilde{U}_G)^{-q}$$

$$\widetilde{U}_G^2\left(\frac{\widetilde{w}_i}{E_G} + \frac{\widetilde{p}_G}{\pi E_G} + \frac{\widetilde{w}_i}{E_L}\right) = 0 \tag{37}$$

where

$$X^2 = \frac{\dfrac{4C_L}{D}\left(\dfrac{U_L{}^S D}{\nu_L}\right)^{-m}\dfrac{\rho_L(U_L{}^S)^2}{2}}{\dfrac{4C_G}{D}\left(\dfrac{U_G{}^S D}{\nu_G}\right)^{-q}\dfrac{\rho_G(U_G{}^S)^2}{2}} = \frac{(\Delta P/L)_L{}^S}{(\Delta P/L)_G{}^S} \tag{38}$$

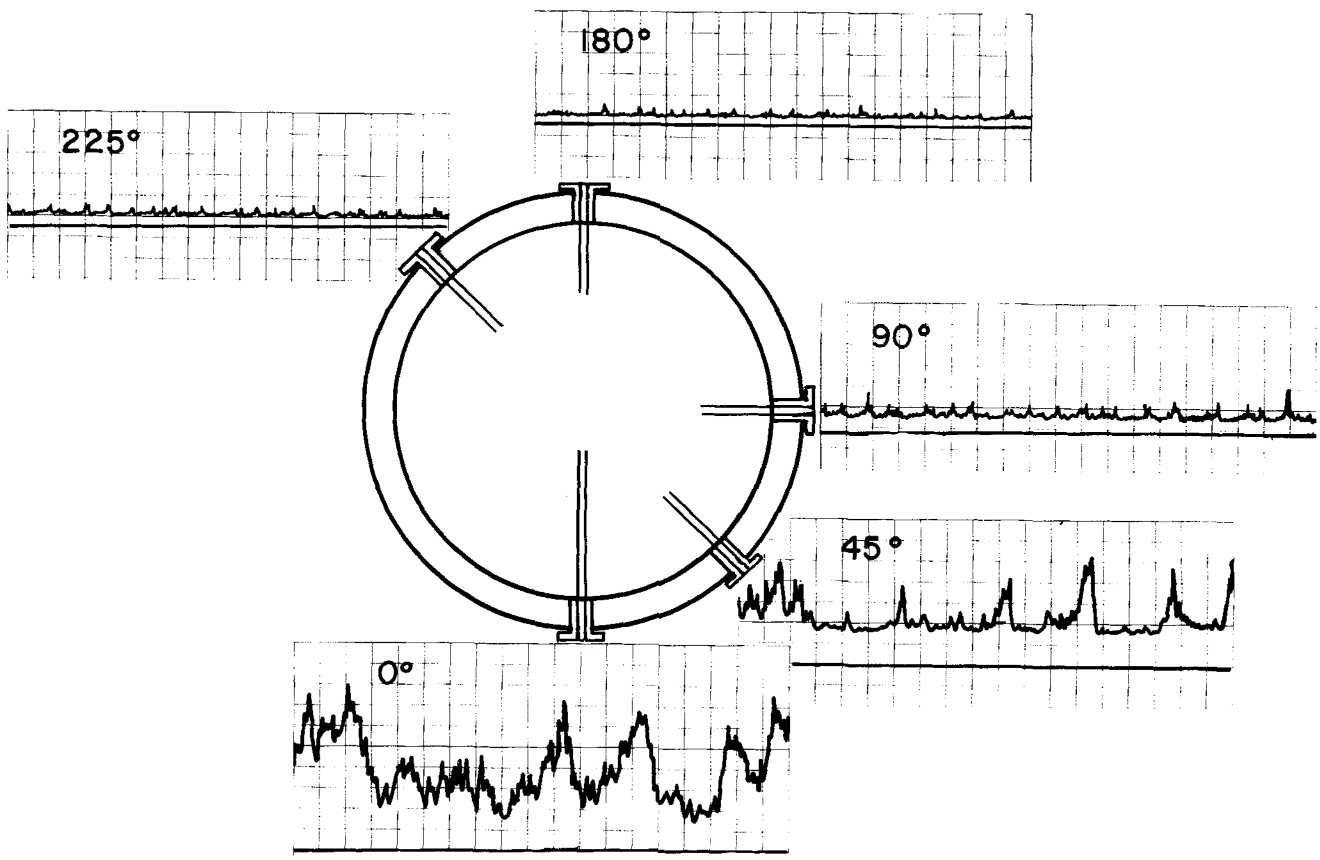

Fig. 7. Typical oscillograph tracings of the interfacial structure at the five circumferential positions.

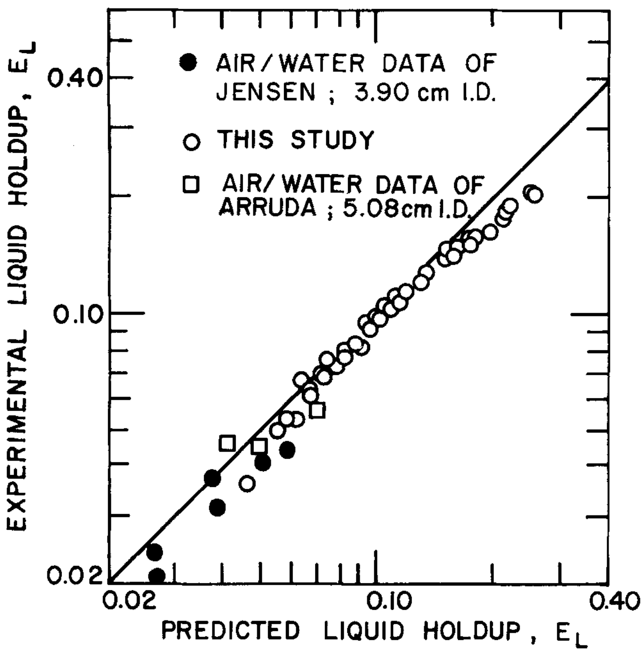

Fig. 8. A comparison between the predicted and experimental values of the liquid holdup for small amplitude interfacial wave conditions.

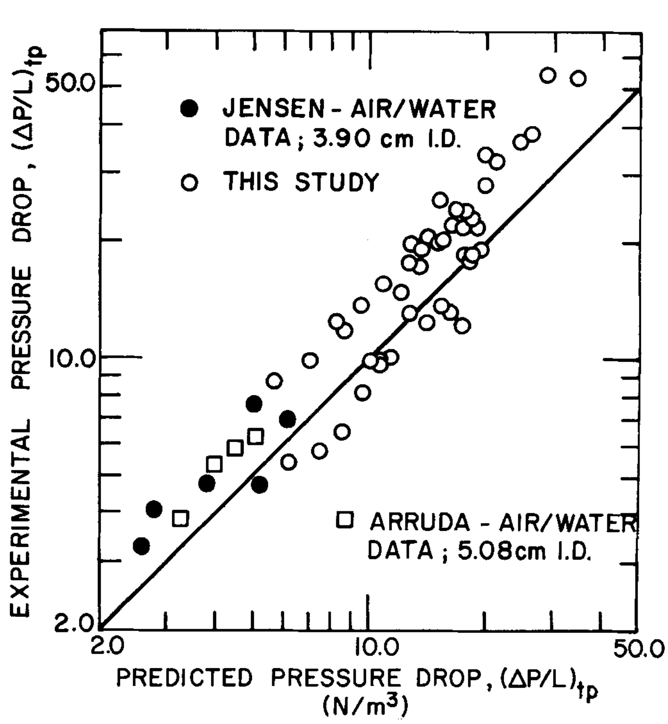

Fig. 9. A comparison between the predicted and experimental values of pressure drop for small amplitude interfacial wave conditions.

and $\tilde{w}_i$, $\tilde{p}_G$, and $\tilde{p}_L$ are given by Equations (2) and (5). The dimensionless equivalent diameters are defined by $\tilde{D}_{EG} = D_{EG}/D$ and $\tilde{D}_{EL}/D$, and

$$\tilde{U}_L = \bar{U}_L/U_L^S = 1/E_L \tag{39}$$

$$\tilde{U}_G = \bar{U}_G/U_G^S = 1/E_G \tag{40}$$

Equation (37) is the same as the results of Johanessen (1972) and Taitel and Dukler (1976) for the holdup as a function of the Lockhart-Martinelli parameter.

Thus, all of the geometrical parameters associated with stratified flow can be related to the Lockhart-Martinelli parameter X using Equation (37). The solution of Equation (37) for E_L as a function of X can be prepared in the form of a plot to provide a trial value for initiating the iterative procedure discussed above. The Lockhart-Martinelli correlation significantly overpredicts the holdup, as we shall show.

THE EXPERIMENTS

The experimental test loop consisted of a clear acrylic pipe 63.5 mm ID and 6 m in length which was cut into several sections, flanged, and mounted on a trestle. The tube was supported every 2 m by rubber cushioned blocks, and it was leveled to within 2 mm.

Air was supplied from a 7.5 hp turbo-compressor and was regulated manually by means of a butterfly valve. The air was saturated and cooled in a spray chamber prior to metering it with a venturi. Downstream of the venturi, tap water was introduced through a liquid distributor which brought the gas and liquid in contact as parallel streams, and the liquid velocity could be varied by adjusting the distributor. The

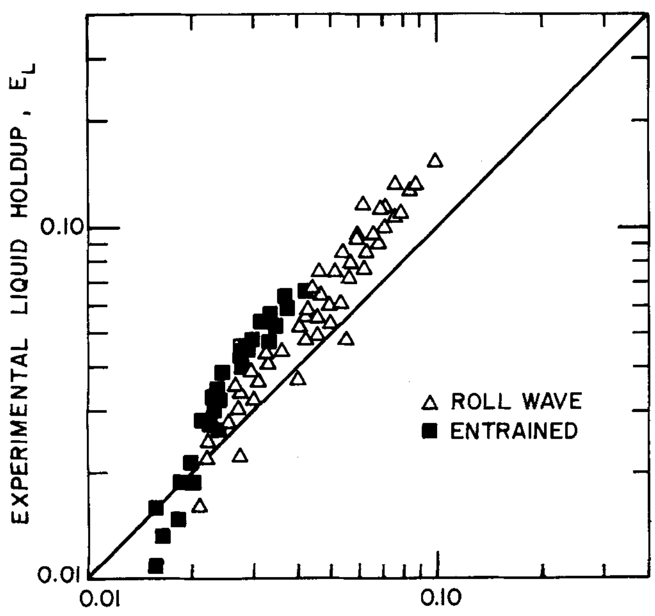

Fig. 10. A comparison between the predicted and experimental values of holdup for roll wave conditions.

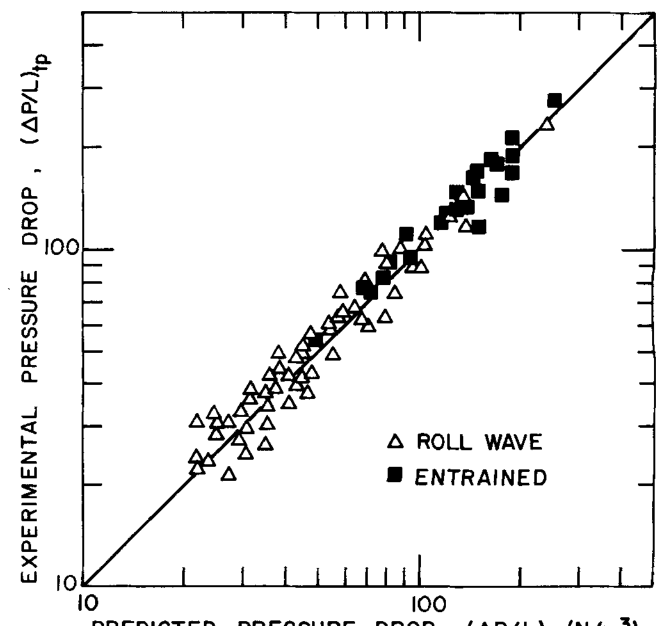

Fig. 11. A comparison between the predicted and experimental values of pressure drop for roll wave conditions.

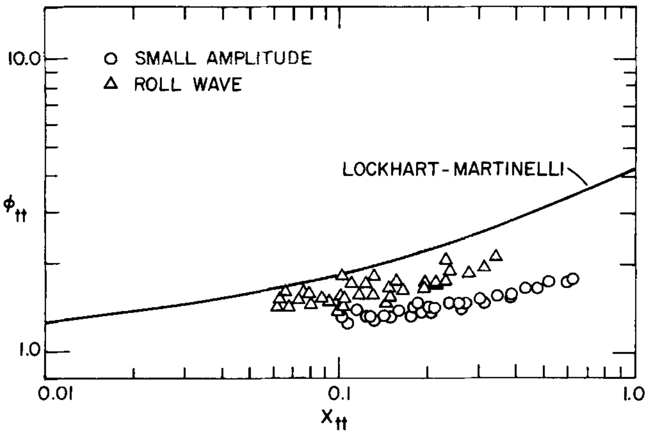

Fig. 12. A comparison of the pressure drop data with the Lockhart-Martinelli correlation.

liquid flow was measured using a calibrated rotameter, and the liquid was recycled by passing the exit streams into a gas-liquid separator. This tank was equipped with baffles and a demister.

The pressure drop was measured by micromanometers connected to pressure taps installed at various axial positions along the top of the pipe. The in situ volume fractions were measured with electrical conductance probes inserted around the periphery of the tube. These probes consisted of parallel wires mounted in plugs that fitted flush with the inside tube surface as shown in Figure 6. The electrical conductance between the probes is proportional to the length of wire immersed in the liquid over the range of liquid layer thicknesses encountered here. A 30 KHz AC signal was supplied to the conductance probe to avoid polarization of the electrodes, and the output was filtered to eliminate the high frequency component but not the low frequency signal associated with interfacial waves. The output was recorded on an oscillograph to obtain the typical results shown in Figure 4 for small and large amplitude waves.

EXPERIMENTAL RESULTS

Table 1 summarizes the range of flow rates studied. The flow regime ranged from low holdup, small amplitude wave conditions to the onset of significant entrainment and dispersed/annular flow. At the higher gas and liquid flow rates, the liquid climbed the wall leading to annular flow, and roll waves predominated. Figure 7 shows oscillograph tracings for probes positioned at various circumferential stations near the onset of annular flow.

Experimental and predicted values of the in situ volume fraction of liquid (holdup) are compared in Figure

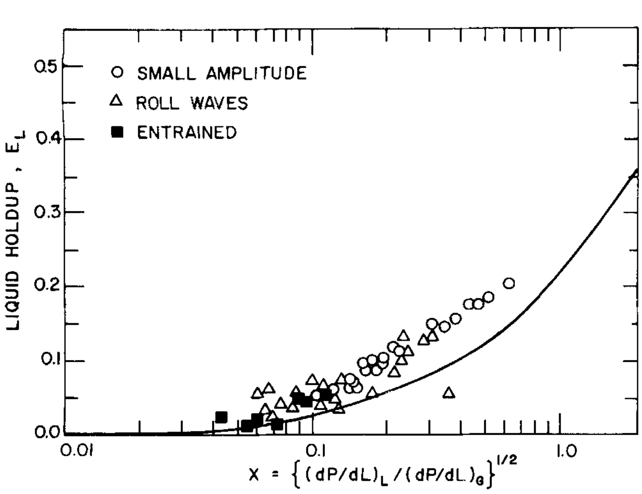

Fig. 14. A comparison between the holdup data and the holdup predicted from Equation (37).

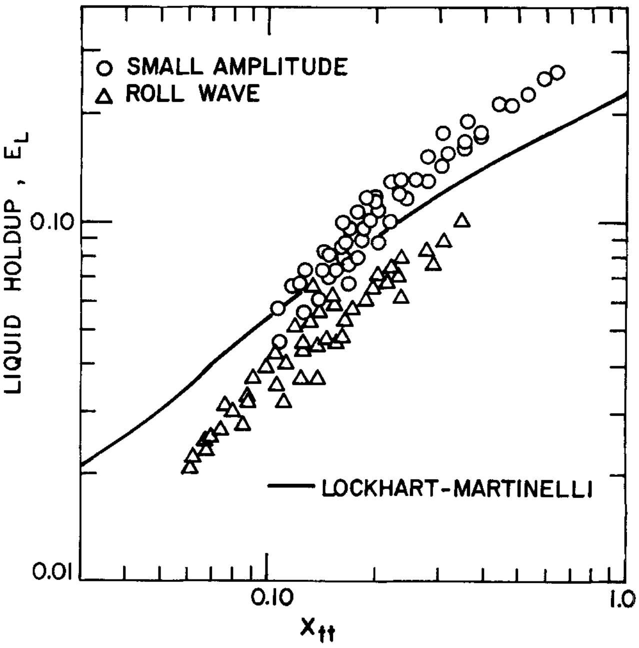

Fig. 13. A comparison of the holdup data with the Lockhart-Martinelli correlation.

8. Included in the figure are some of the results of Arruda (1970) and Jensen (1972) for air/water flow in tubes with inside diameters of 25.4, 39.0, and 50.8 mm. Their data presented in Figure 8 are those that fall in the wavy stratified region of the Baker-Scott (1963) flow map and have gas and liquid Reynolds numbers, based on the cross-sectional area occupied by each phase, greater than 2,100. Most of their results were for laminar/laminar and laminar/turbulent flows, however.

It should be pointed out that Jensen measured y_0 using a micrometer depth gauge mounted on the top of the tube and by means of a two-wire conductance probe similar to ours. Arruda, on the other hand, used quick closing valves to measure the holdup. Their results are

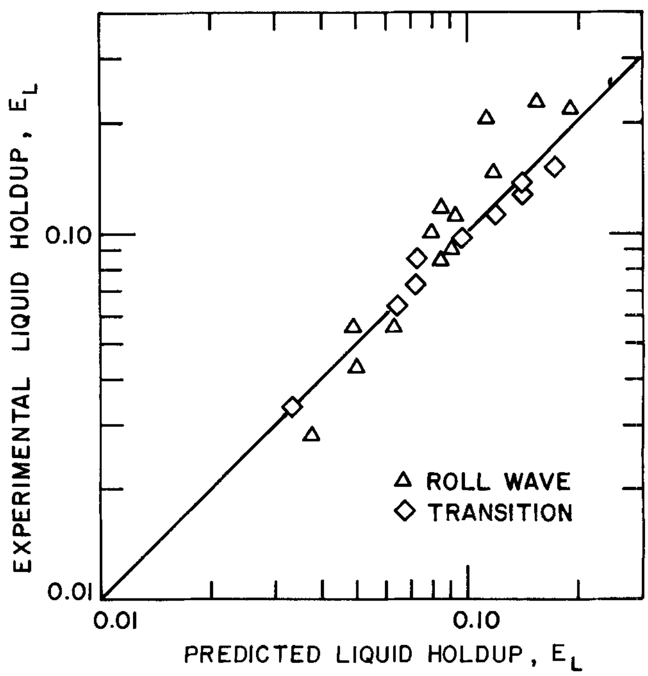

Fig. 15. A comparison between the predicted and experimental values of holdup for roll wave conditions using the interfacial shear equation for small amplitude waves.

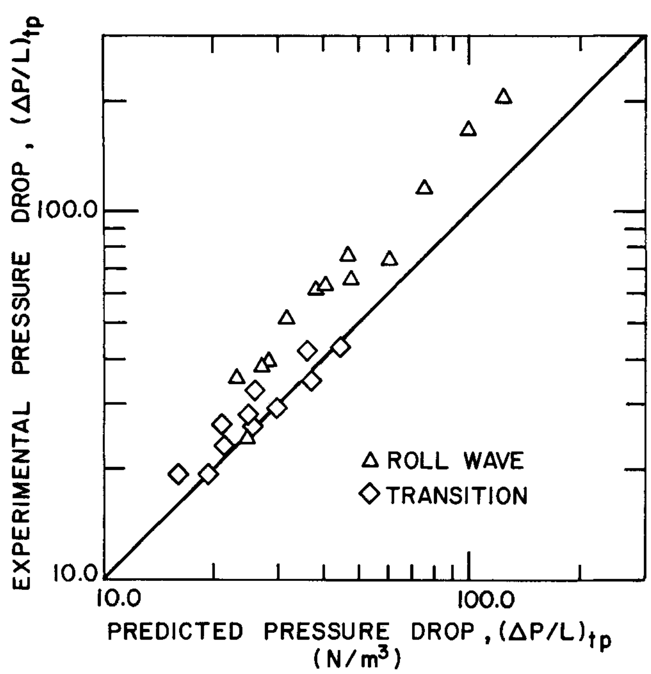

Fig. 16. A comparison between the predicted and experimental values of pressure drop for roll wave conditions using the interfacial shear equation for small amplitude waves.

consistent with the new data, and all of the experimental results are in better agreement with the mathematical model proposed above than with the Lockhart-Martinelli correlation as we shall show.

Since the small amplitude wave regime involves a different equation for τ_i than the roll wave regime, we shall present the results for these regimes separately. Figures 8 and 9 show comparisons between measured and predicted values of holdup and pressure drop for small amplitude waves. The average percent deviation of the holdup for our 62 data points is 7.7%, which is excellent agreement. The predicted results for pressure drop tend to lie below experimental values, and the average deviation is 24.3% for the data of Figure 9. Note in Figure 8 that as the experimental liquid holdup increases (that is, at conditions of lower gas flow rate), predicted values begin to deviate substantially from measurements. This is not surprising, as we have imposed a severe limitation on the model for the liquid phase by assuming $\tau/\tau_{WL} \simeq 1$ [Equation (13)]. This approximation becomes highly questionable at larger values of the holdup.

Figures 10 and 11 show the results of experiment and mathematical models for roll wave conditions. The predicted pressure drop results scatter around the experimental values, and the predictions are unexpectedly good. The average deviation is only 4.6% for the 111 data points of Figure 11. The holdup results shown in Figure 10 are more scattered than for small amplitude wave conditions, and the data are underpredicted at larger E_L (again due to our assumption $\tau/\tau_{WL} \simeq 1$). The average deviation is 26.4% for 73 data points. The scatter in the holdup results is due primarily to the uncertainty associated with determining a meaningful average liquid level in the tube, for the large amplitude roll waves have irregular frequencies and amplitudes. It is of importance to note that our definition for $\overline{\delta}$ [Equation (8)] from which experimental values of E_L are computed is based on a time-average expression which may not adequately describe roll waves. A spatial average for $\overline{\delta}$ usually produces higher values of experimental E_L which would lead to poorer agreement between predictions and experiments.

The predictions of the analysis proposed here are superior to the Lockhart-Martinelli correlations for pressure drop and holdup, which are defined on the basis of superficial Reynolds numbers. Figure 12 is a comparison of the Lockhart-Martinelli correlation for pressure drop and the experimental results for large and small amplitude wave conditions. The predictions are particularly bad for small amplitude waves, for $(\Delta P/L)$ is overpredicted by over 100% for larger values of E_L. As shown in Figure 13, there is better agreement between predicted and experimental values of E_L than for $(\Delta P/L)$, but the experimental data increase with an increase of X_{tt} at a greater rate than the correlation predicts.

The modified Lockhart-Martinelli approach developed here, Equation (37), which was used to provide trial values of E_L for the iterative procedure discussed above is in better agreement with the holdup data than the original correlation. Comparison of the data with the modified correlation is shown in Figure 14, but Equation (37) provides useful starting values for the iterative method.

It should be pointed out that the predictions of the model are not too sensitive to the equations used to calculate the interfacial shear in the region near the transition from small amplitude waves to roll waves, for Equations (24) and (25) with (26) predict similar values of τ_i in the intermediate regime between smooth interfaces and the onset of entrainment. Figure 15 shows the results using Equation (25), which should apply only to small amplitude waves, to predict the pressure drop for roll wave conditions near the transition. The results are not significantly different from those presented in Figure 11, and the predicted holdup is actually better, as shown in Figure 16.

A severe limitation of the model is the dependency on the empirical expressions for f_i [Equations (23) and (26)]. The Hanratty et al. expressions are based on air-water data only and have not been shown to apply to other systems. In general, friction factor data for roll wave conditions have not been well correlated, and few relations appear in the literature. By using Equation (26) we have extrapolated the use of the expression to the range of Re_L in this study with the justification being that it does a reasonable job in predicting pressure drop (Figure 11). (It should be pointed out that other friction factor expressions were examined in addition to a simpler analysis for the liquid phase by using expressions similar to the gas phase. Significantly poorer agreement between experiment and predicted values for E_L and $(\Delta P/L)_{tp}$ was observed.

The model's success is in part attributed to the use of the proper f_i for the specific interfacial disturbance encountered. In Figure 16 we note that a slightly worse prediction of the two phase pressure drop is realized for roll waves by using a different expression for f_i [Equation (23) for small amplitude waves]. This is further evidence that we are correct in accounting for the nature of the interfacial structure in f_i and in applying these expressions.

The model presented is a significant improvement over the Lockhart and Martinelli correlations for pressure drop and holdup. A significant shortcoming with the various other semiempirical models sited is that they do not form a basis for predicting heat transfer characteristics, which the present treatment does by making it possible to apply analogies between momentum transfer and heat transfer.

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NOTATION

A = cross-sectional area of flow, m^2
c = wave velocity, m/s
C = wave velocity, m/s
C_G, C_L = constants in Equation (38)
D = tube diameter, m
D_{EG}, D_{EL} = equivalent diameters for gas and liquid phases, respectively, m
E = in-situ volume fraction
f = friction factor
p = perimeter of the tube, m
$(\Delta P/L)$ = pressure drop per unit length, Pa/m
Q = volumetric flow rate, m^3/s
R = tube radius, m
R^+ = dimensionless tube radius
Re = Reynolds number
$\overline{U}_B$ = average gas velocity over the region between the interface and the position of the maximum gas velocity, m/s
U = velocity, m/s
$u^+ = u/u^*$ = dimensionless velocity
$u^* = (\tau_{WL}/\rho_L)^{1/2}$ = friction velocity, m/s
$u_o^* = (\tau_i/\rho_G)^{1/2}$ = friction velocity, m/s
$\overline{U}$ = average velocity, m/s
w_i = width of the interface, m
W = mass flow rate, kg/s
X = Lockhart-Martinelli parameter
y = distance from the tube wall in the direction of the inward normal
y_o = distance from the tube bottom vertically to the interface
$y^+ = yu^*/\nu$ = dimensionless distance

Greek Letters

Γ = volumetric flow per unit width of parallel plate channel, m^2/s
γ = angle subtended by the liquid wetted perimeter
$\overline{\delta}$ = average depth of the liquid layer, m
δ_c = height of most commonly observed intermediate waves
δ_{max} = maximum height of reproducible waves
δ_{min} = thickness of base film
ϵ_M = eddy viscosity, m^2/s
θ = angle measured from the tube bottom
λ_o = friction factor
μ = viscosity, kg/m-s
ν = kinematic viscosity, m^2/s
ρ = density, kg/m^3
τ = shear stress, N/m^2
χ = a constant in Equation (18)

Subscripts

G = gas phase
i = interface
L = liquid
tp = two phase flow
W = wall

Superscripts

S = single phase flow
— = average value
$+, \sim$ = dimensionless quantities

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